RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2015

SECOND YEAR

Date : 22/05/2015 **ECONOMICS (Honours)**

Time : 11 am – 3 pm **Paper :** IV Full Marks : 100

[Use a separate Answer book for each group]

Group – A

1. Answer **any three** questions of the following : (3 × 4)
   a) Discuss the product exhaustion theorem.
   b) Show that price of an exhaustible resource depend on interest rate and total output.
   c) When is an allocation said to be a Pareto Preferred allocation compared to another? When is an allocation Pareto optimal? (2 + 2)
   d) Explain, with the help of a diagram the second fundamental theorem of welfare economics.
   e) What is certainty equivalent of a lottery?
   f) Suppose an individual faces a choice as described below :

	State 1	State 2
Option A	100	100
Option B	200	50
Probabilities	1/3	2/3

   Show that a risk-averse individual will prefer A.

2. Answer **any one** question of the following : (1 × 8)
   a) Consider a two persons two commodity pure exchange economy with $U_1=q_{11}^{\alpha}\, q_{12}$ and $U_2=q_{21}^{\beta}\, q_{22}$, $q_{11}+q_{21}=q_1^0$ and $q_{12}+ q_{22}=q_{\,2}^{0}$. Derive the contract curve as an implicit function of q_{11} and q_{12}.
   b) i) Show that a risk-averse person will prefer the expected value of the lottery with certainty rather than the lottery itself.
      ii) Your utility function is $u=y^{1/2}$ and your initial income is 100. You are offered a lottery where you could win 44 with probability 2/3 and would lose 19 with probability 1/3. Would you take the lottery?

3. Answer **any two** questions of the following : (2 × 15)
   a) i) Discuss the issue of exploitation in imperfectly competitive labor market. (5)
      ii) Analyse the role of trade union in countering exploitation – (A) When labor market is faced by monopolistic exploitation and (B) When labor market is faced by monopsonistic exploitation. (10)
   b) i) Explain the process of representing Pareto optimum allocations in both production and consumption sectors. (12)
      ii) Explain the concept of Grand Utility Possibility frontier in brief. (3)
   c) i) Consider an individual with initial wealth equals to Rs. 1 lakh faces a risk of complete loss with probability p = 0.01. If the insurance company charges a premium rate 0.02, find the optimum level of insurance if $u=y^{1/2}$. (6)
      ii) Construct a situation where sellers of a product are better informed about the quality of a product than the buyers. Explain how asymmetric information of this type can create market failure in which bad products drive out good products from the market. (6 + 3)
      Suggest some strategies to eliminate this type of market failure.

d) i) How do you define (I) absolute risk aversion and (II) relative risk aversion?

ii) Solve for the absolute risk aversion coefficient and relative risk aversion co-efficient for the quadratic utility function

$U(x) = a + bx + cx^2$; b, c > o, and show that absolute risk aversion is increasing at any level of x.

iii) Consider the following utility functions of individual A and B, $U(w)_A = -e^{-aw}$, $U(w)_B = -e^{-bw}$, with a > b. Calculate Arrow Pratt. Absolute Risk Aversion Coefficient for both individuals and comment on their attitude towards risk.

Group – B

4. Answer **any four** questions of the following : (4 × 5)

a) Consider the model $Y_i = \beta X_i + u_i, \quad i = 1, 2, ..., n.$ Where u_i is a stochastic variable with constant variance σ^2 & expectation zero and assume that X is non-stochastic.

Show that the estimators $\hat{\beta} = \frac{\sum X_i Y_i}{\sum X_i^2}$ & $\tilde{\beta} = \frac{\sum Y_i}{\sum X_i}$ are both unbiased estimators of β & show $\text{var}(\hat{\beta}) < \text{var}(\tilde{\beta})$.

b) For 20 army personnel, the regression of weight of kidneys (y) on weight of heart (x), both measured in oz., is y = 0.399x + 6.934 and the regression of weight of heart on weight of kidney is x = 1.212y-2.461.

What percentage of variation in the weight of kidney, on an average, is explained by the weight of heart?

c) State clearly the assumptions in CLRM. In this connection, state the Gauss-Marcov theorem.

d) Describe the construction of Durbin-Watson statistic.

e) Consider the following regression :

$\hat{Y}_i = 0.2033 + 0.6560 X_t$

$SE = (0.0976) \quad (0.1961)$

$r^2 = 0.397 \qquad RSS\,(\text{Residual } SS) = 0.0544 \qquad ESS\,(\text{Explained } SS) = 0.0358$

Where Y = labour force participation on rate (LFPR) of women in 1972 and X = LFPR of women in 1968. The regression results were obtained from a sample of 19 cities in USA. (2 + 3)

i) How do you interpret the regression?

ii) Test the hypothesis $Ho: \beta_2 = 1$ against $H_1: \beta_2 > 1$ at 5% level of significance

[Given $t_{0.05,17} = 1.74$]

f) In the context of simple linear regression between y & x (y : dependent & x : independent), show that the coefficient of determination coincides with the squared coefficient of correlation between y & x.

g) Consider the following regression model : (2 + 3)

$$\frac{1}{Y_i} = \beta_1 + \beta_2\left(\frac{1}{X_i}\right) + u_i$$

Where neither X nor Y assume zero value.

i) Is this a linear regression model?

ii) How would you estimate this model?

5. Answer **any two** questions of the following : (2 × 15)

A) a) A researcher at ISI, calcutta is conducting a study about tuition at West Bengal Colleges. So far, he has collected data from 20 colleges about their tuition costs, Y (in thousands of rupees), their score on an independent rating scale, X_1 (in points out of 100), their size X_2 (in thousands of under graduates). A print out of the regression of Y (tuition) on rating (X_1) & size (X_2) is shown below. Use it to answer the questions below :

[Assume the population model : $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$ & u_i's are iid $N(0, \sigma^2)$],

The regression equation is :

Tuition = -2.40 + 0.09 Rating -0.01 size.

Regressors	Coeff	Stderror	t
Constant	-2.40	0.93	-2.6
Rating	0.09	0.02	4.25
Size	-0.01	0.01	-1

ANOVA

Source	df	SS
Regression	2	1742
Residual Error	17	5
Total	19	1747

$R^2 = 99\%$

i) The author of the study wants to know whether the combination of rating & size is overall useful for predicting tuition costs. Perform an appropriate hypothesis to answer this question (Given $F_{0.05, 2, 17} = 3.59$)

ii) Use the printout to compute a 95% confidence interval for β_2 [Given $t_{0.025, 17} = 2.11$]

b) In the context of heteroskedasticity, describe the Goldfeld – quandt test. (4 + 4 + 7)

B) A sample of data consists of n observations on two variables X & Y. The model is

$Y_i = \beta_1 + \beta_2 X_i + \in_i$ .... (1)

Where β_1 and β_2 are parameters & $\in$ is a disturbance term that satisfies the usual regression model assumptions.

In view of the true model, $\bar{Y} = \beta_1 + \beta_2 \bar{X} + \bar{\in}$ .... (2),

Where $\bar{Y}, \bar{X} \& \bar{\in}$ are sample means of Y, X & $\in$. Subtracting the second equation from the first, one obtains

$Y_i^* = \beta_2 X_i^* + \in_i^*$ .... (3), where $Y_i^* = (Y_i - \bar{Y}), X_i^* = (X_i - \bar{X})$ & $\in_i^* = \in_i - \bar{\in}$

One researcher fits $\hat{Y} = b_1 + b_2 X$ ....(4) & another researcher fits $\hat{Y}^* = b_1^* + b_2^* X^*$ .... (5) [Note : The second researcher included an intercept in the specification]

a) Comparing regressions (4) & (5) , demonstrate $b_2^* = b_2$ & $b_1^* = 0$ [Explicitly derive the expressions using OLS]

b) Show $\hat{Y}_i^* = (\hat{Y}_i - \bar{Y})$.

c) Show that the residuals in regression equation (5) are identical to the residuals in (4). (8 +4 + 3)

C) Assume the model $Y_t = \alpha + \beta X_t + \in_t$, where $E(\in_t) = 0$; $v(\in_t)$ is constant but $\text{cov}(\in_t, \in_s) \neq 0$ for $t \neq s$.

Further assume $\in_t = \rho \in_{t-1} + u_t$, Where $E(u_t) = 0$ & $V(u_t) =$ constant & $E(u_t u_{t-1}) = 0$.

Now show $\hat{\beta}$ [OLS estimator of β] is unbiased.

Also show that variance of $\hat{\beta}$ is no longer the least [assume ρ to be positive]. (5 + 10)

D) Can you estimate the parameters of the models

$|\hat{u}_i| = \sqrt{\beta_1 + \beta_2 X_i} + V_i$

$|\hat{u}_i| = \sqrt{\beta_1 + \beta_2 X_i^2} + V_i$

by the method of ordinary least squares?

a) Why or why not?

b) If not, can you suggest a method, informal or formal, of estimating the parameters of such models? (7 + 8)